GCSE Maths – Algebra

Gradients and Areas of Graphs in Context (Higher Only)

Worksheet

NOTES

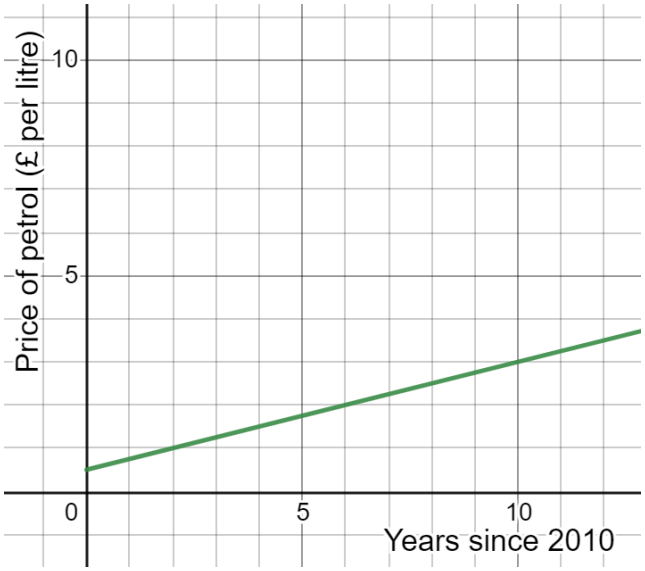
SOLUTIONS

This worksheet will show you how to work out different types of questions relating to gradients and areas of graphs in. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Below is a graph of the change in the price of petrol over time. What rate is the price of petrol increasing at?



Step 1: Work out if the graph is linear or non-linear.

This graph is linear as the line is straight. This makes finding the gradient of the line (which represents the rate of change of the price of petrol) easier.

Step 2: Identify two points on the line, then calculate the gradient by calculating the difference in y divided by the difference in x.

We can use the points $(2, 1)$ *and* $(10, 3)$.

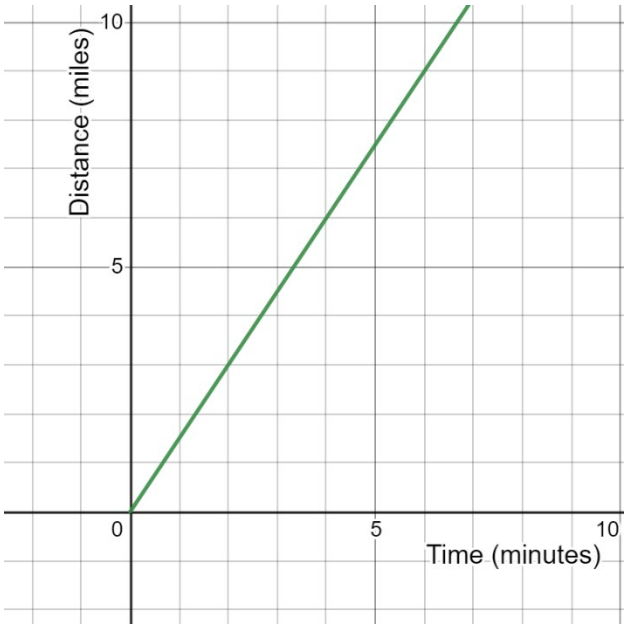
$$Gradient = \frac{Change\ in\ y}{Change\ in\ x} = \frac{3-1}{10-2} = \frac{2}{8} = \frac{1}{4}$$

Step 3: Interpret the gradient.

The gradient is $\frac{1}{4}$.
This means that each year, the price of petrol increases by $\frac{1}{4}$ *of* $£1$, *or* $25p$, *per year.*

Guided Example

Below is a distance-time graph of a car. What is the speed of this car?



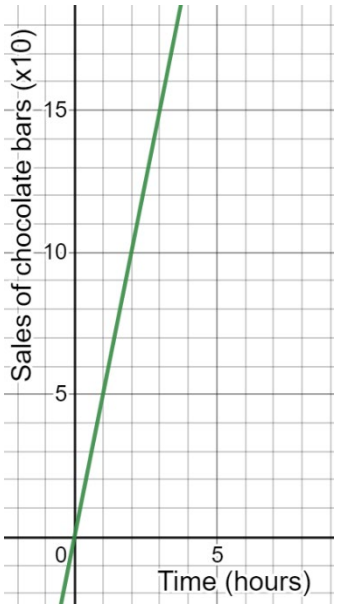
Step 1: Work out if the graph is linear or non-linear.
.

Step 2: Identify two points on the line, then calculate the gradient by calculating the difference in y divided by the difference in x.

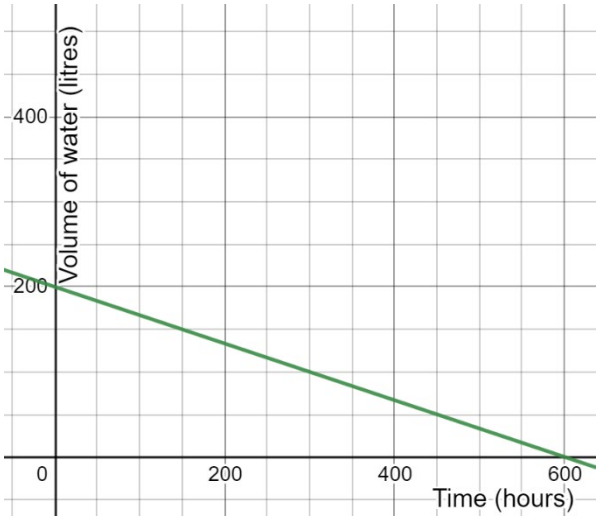
Step 3: Interpret the gradient.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

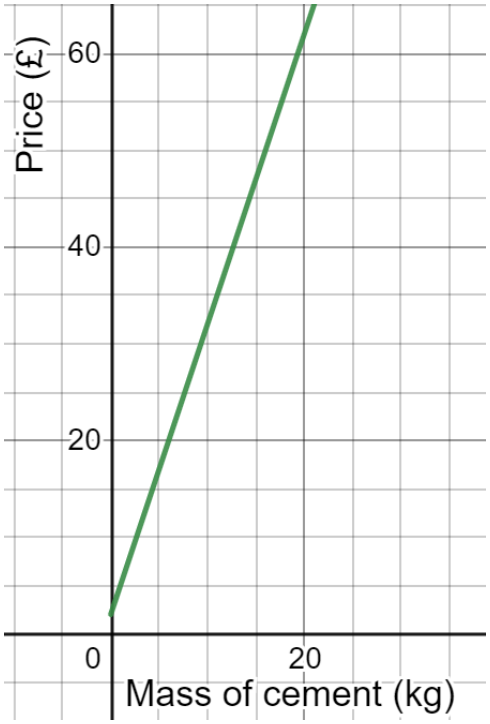
1. Calculate the rate of sales of chocolate bars.



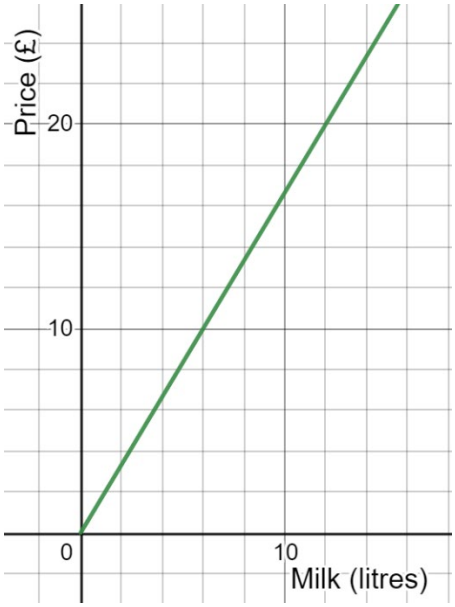
2. Calculate the rate of water being emptied from a tank



3. Calculate the price per kg of cement



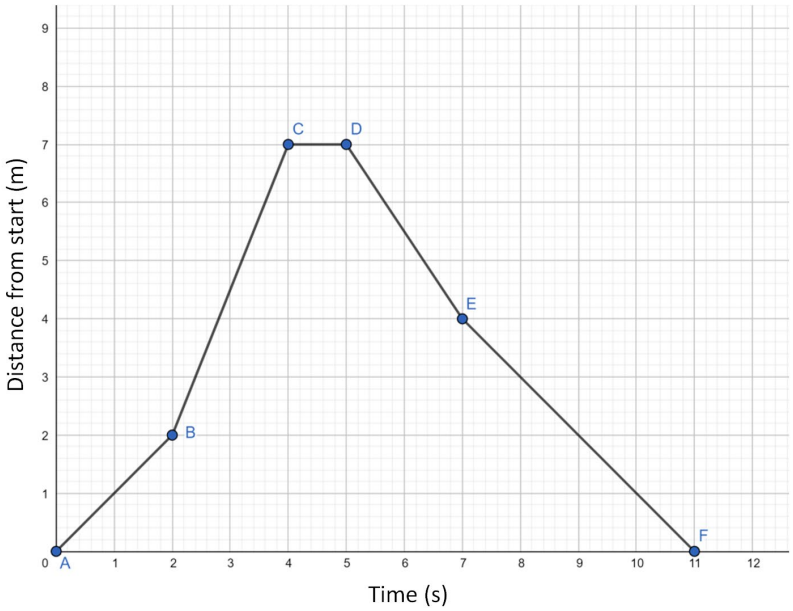
4. Calculate the price per pint of milk.



Section B

Worked Example

Below is a displacement-time graph of an object.
Calculate the speed of the object between points B and C.



Step 1: Work out the coordinates of the points asked for.

Point B is at $(2, 2)$ *and point C is at* $(4, 7)$.

Step 2: Calculate the gradient of the line between the required coordinates.

To find speed, we need to divide the total distance by the time. Therefore, we divide the difference in y by the difference in x.

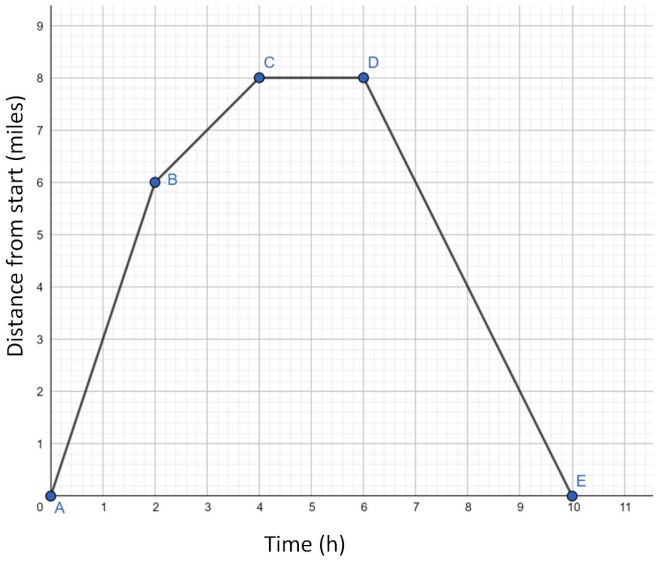
$$Gradient = \frac{Change\ in\ y}{Change\ in\ x} = \frac{7-2}{4-2} = \frac{5}{2}$$

Step 3: Interpret the gradient in terms of the question context.

The gradient tells us the speed of the object between points B and C. The speed is $2.5\ m/s$.

Guided Example

The displacement-time graph below shows the distance walked by a group. Calculate their speed between points D and E.



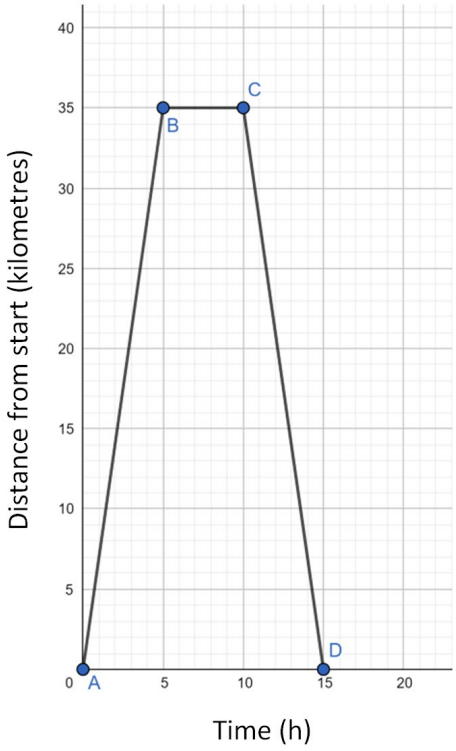
Step 1: Work out the coordinates of the points asked for.

Step 2: Calculate the gradient of the line between the required coordinates.

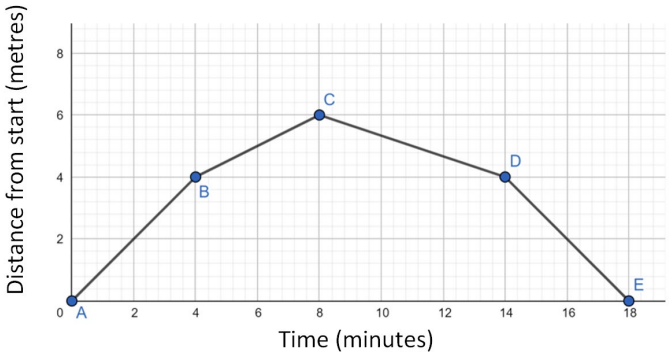
Step 3: Interpret the gradient in terms of the question context.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

5. Calculate the speed of this car between points A and B. What is happening between points B and C?



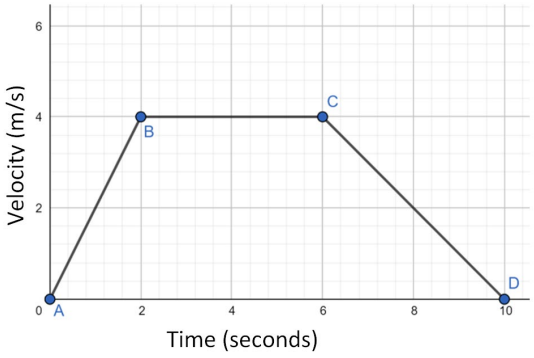
6. Calculate the speed between the points C and D.



Section C

Worked Example

Below is a velocity-time graph of an object. Calculate the acceleration of the object between points A and B, and calculate the total distance travelled by the object.



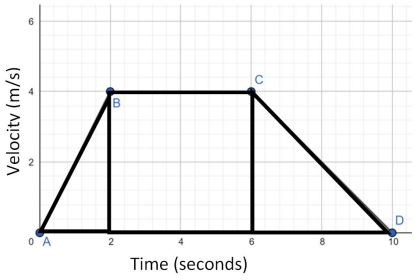
Step 1: Identify the points asked for in the question, then calculate the gradient of the line between these two points.

Point A is at $(0, 0)$ *and point B is at* $(2, 4)$*. The gradient of the line represents acceleration because it is the change in velocity divided by the time it takes.*

$$Acceleration = \frac{Change\ in\ velocity}{Change\ in\ time} = \frac{4-0}{2-0} = \frac{4}{2} = 2$$

The acceleration of the object between points A and B is $\mathbf{2\ m/s^2}$.

Step 2: For the second part of the question, divide the area under the curve into shapes, such as squares, triangles and rectangles.



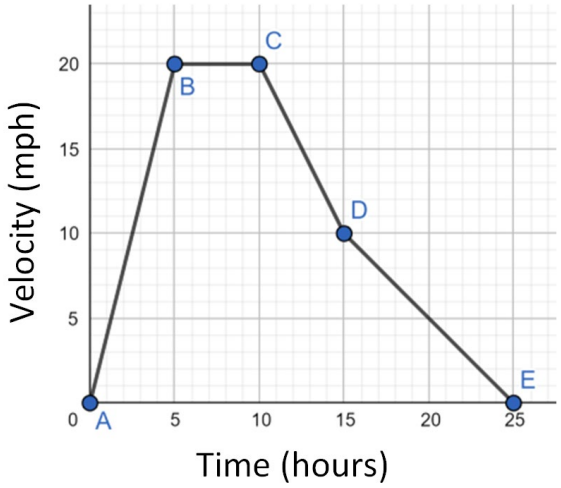
$$First\ Triangle\ Area: \frac{1}{2} \times 2 \times 4 = 4\ \mathrm{m}^2$$
$$Square\ Area: 4 \times 4 = 16\ \mathrm{m}^2$$
$$Second\ Triangle\ Area: \frac{1}{2} \times 4 \times 4 = 8\ \mathrm{m}^2$$

Step 3: Add together the areas of the shapes to find the total distance travelled.

$$Distance\ travelled = Area\ under\ graph = 4 + 16 + 8 = \mathbf{28\ m}$$

Guided Example

Below is a velocity-time graph for a car. Calculate the deceleration of the car between points C and D, and find the total distance travelled by the car.



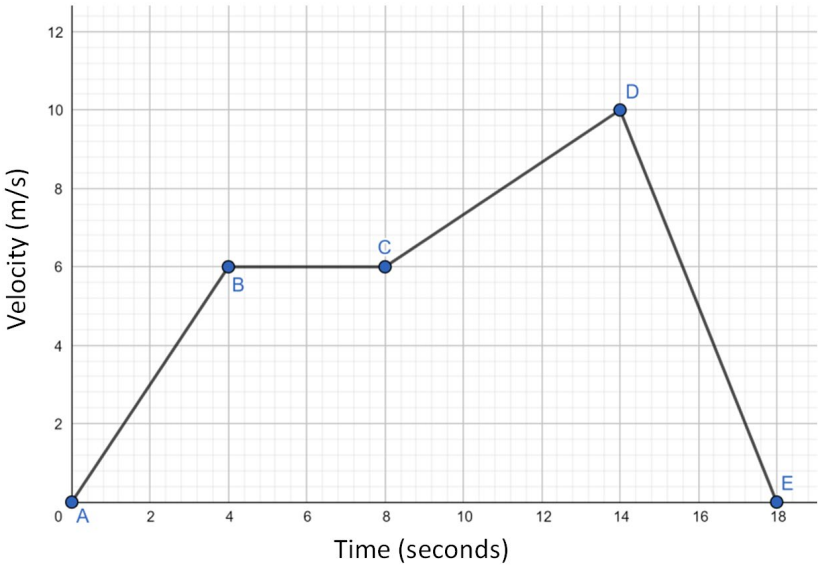
Step 1: Identify the points asked for in the question, then calculate the gradient of the line between these two points.

Step 2: For the second part of the question, divide the area under the curve into shapes, such as squares, triangles, and rectangles.

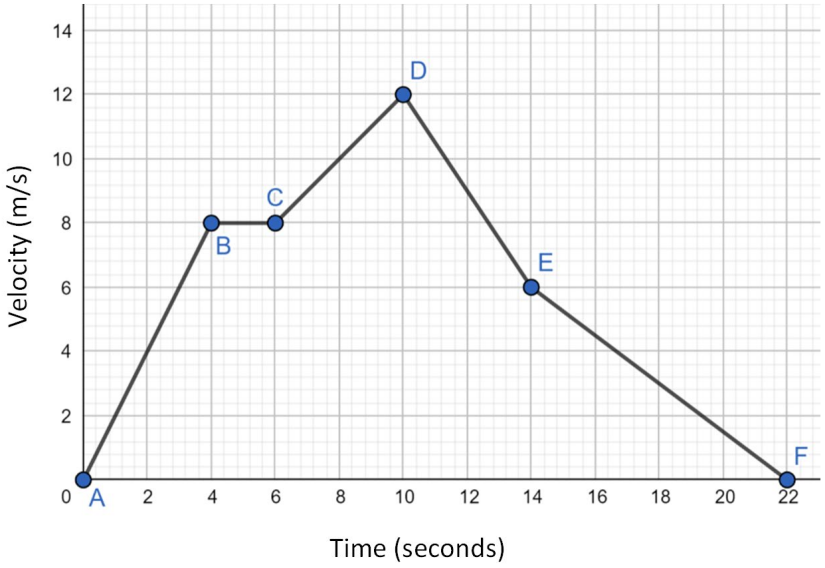
Step 3: Add together the areas of the shapes to find the total distance travelled.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

7. Find the deceleration between points D and E, and calculate the total distance travelled by this object.



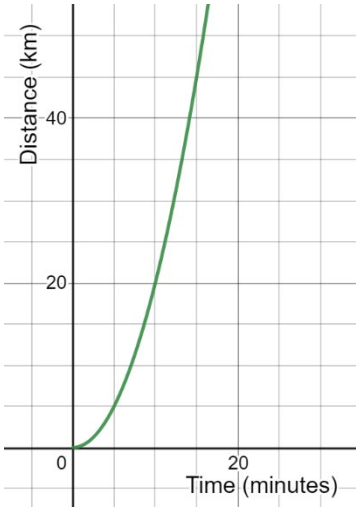
8. Calculate the acceleration between points A and B, and the total distance travelled.



Section D

Worked Example

Below is a distance-time graph of a car. Calculate the speed at which the car is travelling at minute 15.

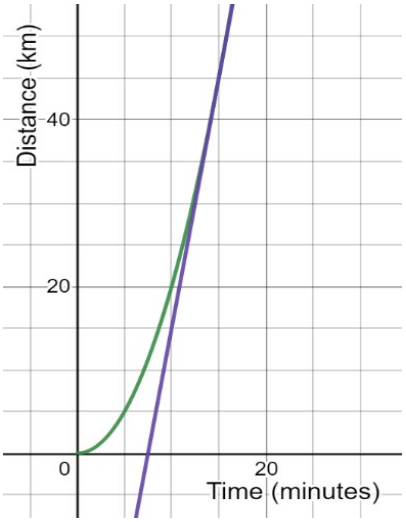


Step 1: Work out if the graph is linear or non-linear.

This graph is non-linear because it is not a straight line. This means we will have to use tangents to find the gradient.

Step 2: Draw a tangent to the line at the point asked about in the question and identify two coordinates on this tangent that could be used to calculate the gradient.

We are asked to find the gradient when $x = 15$. *The point on the line is* $(15, 45)$.



This tangent line passes through the points $(10, 15)$ *and* $(15, 45)$. *Use the difference between the y-coordinates and x-coordinates to find the gradient.*

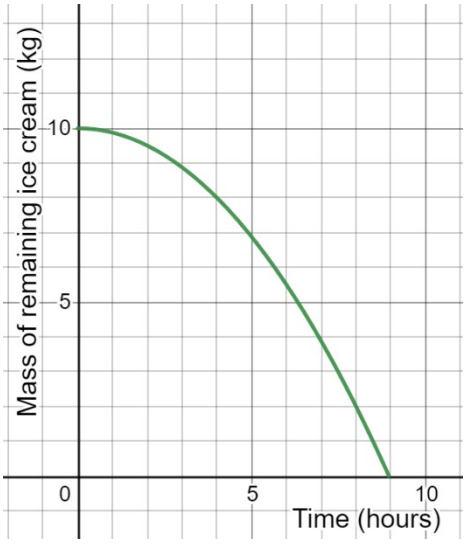
$$Gradient = \frac{Change\ in\ y}{Change\ in\ x} = \frac{45 - 15}{15 - 10} = \frac{30}{5} = 6$$

Step 3: Interpret the gradient in terms of the question context.

The gradient tells us the speed of the car at minute $\mathbf{15}$ *is* $\mathbf{6\ km/min}$.

Guided Example

Below is a graph showing the remaining amount of ice cream in a café over time. Find the rate at which ice cream is being sold at the 4th hour.



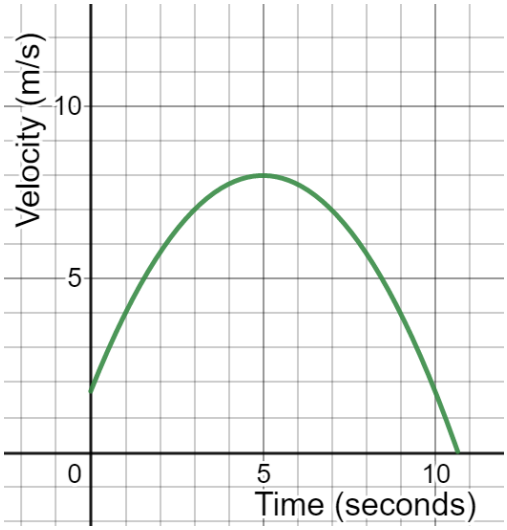
Step 1: Work out if the graph is linear or non-linear.

Step 2: Draw a tangent to the line at the point asked about in the question and identify two coordinates on this tangent that could be used to calculate the gradient.

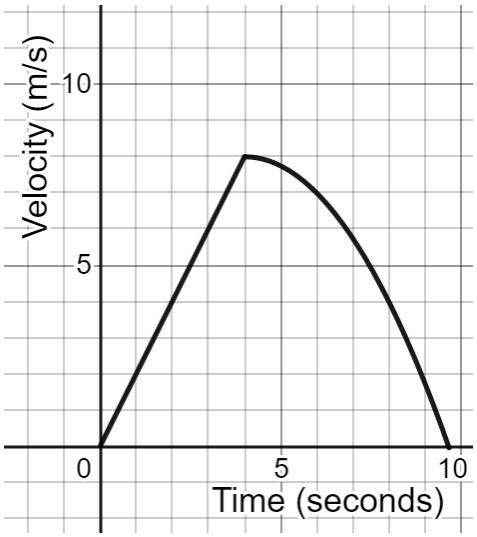
Step 3: Interpret the gradient in terms of the question context.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

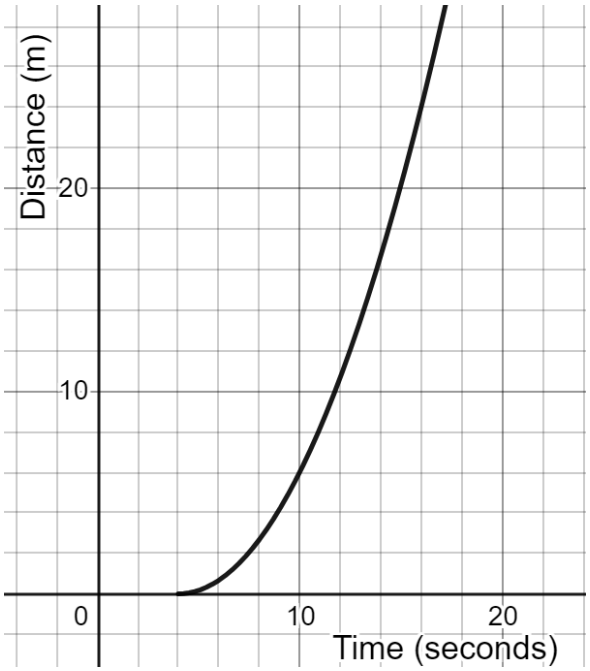
9. The graph below is a velocity-time graph of an object. Find the deceleration of the object at 7 seconds.



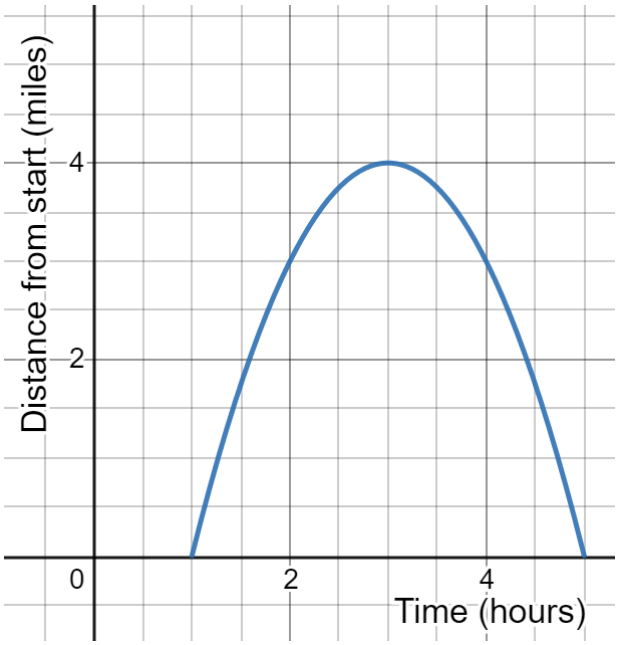
10. Below is a velocity-time graph of an object. Find the deceleration of the object at 6 seconds.



11. Below is a distance-time graph of an object. Calculate the speed at the 14th second.



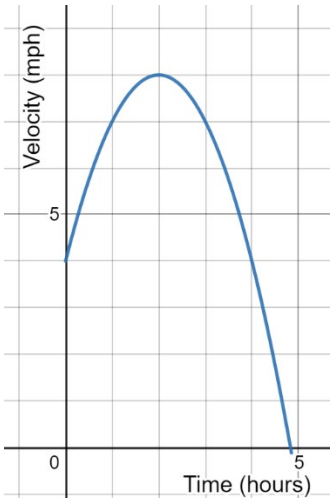
12. Below is a displacement-time graph. Calculate the speed at 3 hours.



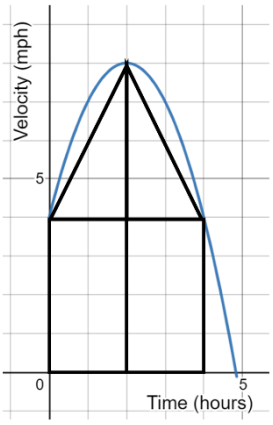
Section E

Worked Example

Below is a velocity-time graph of a car. Estimate the total distance travelled by the car.



Step 1: Draw shapes under the curve to roughly fit the area.



Use squares, rectangles, and triangles to make a rough approximation of the area.

Step 2: Calculate the area of each shape.

$$Rectangles\ Total\ Area = 2 \times (2 \times 4)\text{m}^2 = 2 \times 8\ \text{m}^2 = 16\ \text{m}^2$$

$$Triangles\ Total\ Area = 2 \times (\frac{1}{2} \times 2 \times 4)\text{m}^2 = 2 \times 4\ \text{m}^2 = 8\ \text{m}^2$$

Step 3: Add together the areas of each shape.

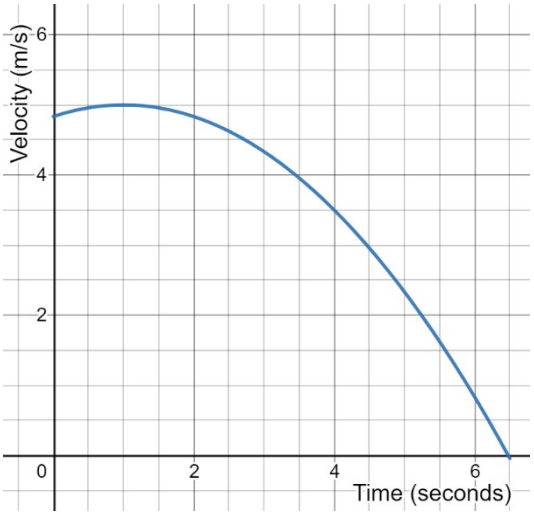
$$Area\ Under\ Curve = 16\ \text{m}^2 + 8\ \text{m}^2 = 24\ \text{m}^2$$

Step 4: Interpret the area in terms of the question context.

The area under the curve is equal to the distance travelled. So, the car travelled **24** *miles in total.*

Guided Example

Below is a velocity-time graph for an object. Estimate the total distance travelled by the object.



Step 1: Draw shapes under the curve to roughly fit the area.

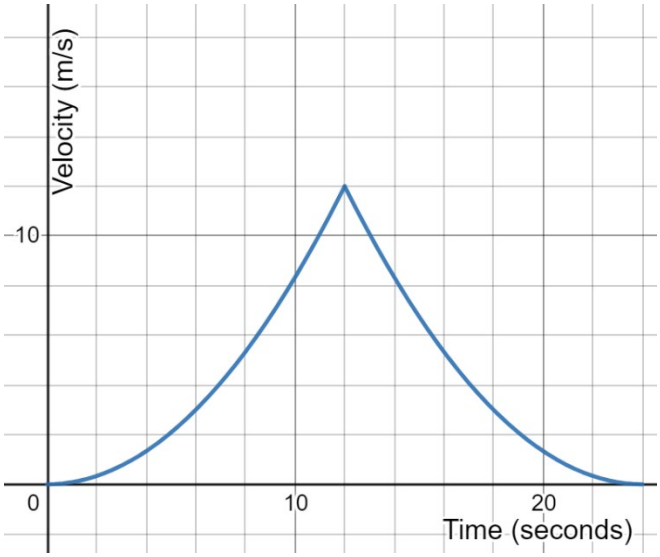
Step 2: Calculate the area of each shape.

Step 3: Add together the areas of each shape.

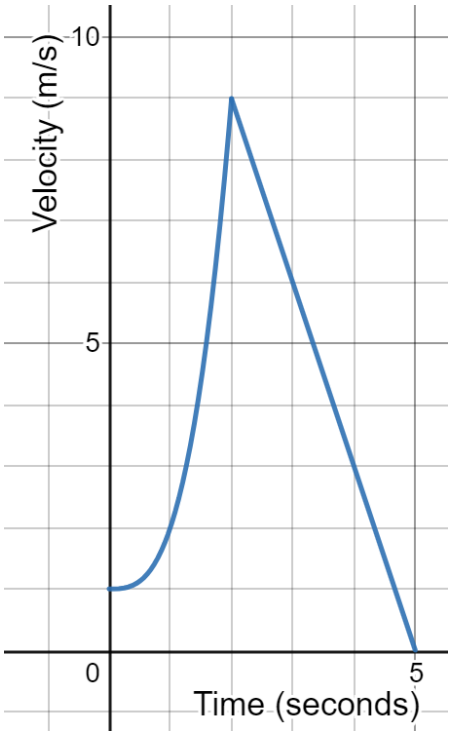
Step 4: Interpret the area in terms of the question.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

13. Calculate the total distance travelled by the object in this graph:



14. Calculate the total distance travelled by the object in this graph:



15. Find the total distance travelled by the object in this graph: